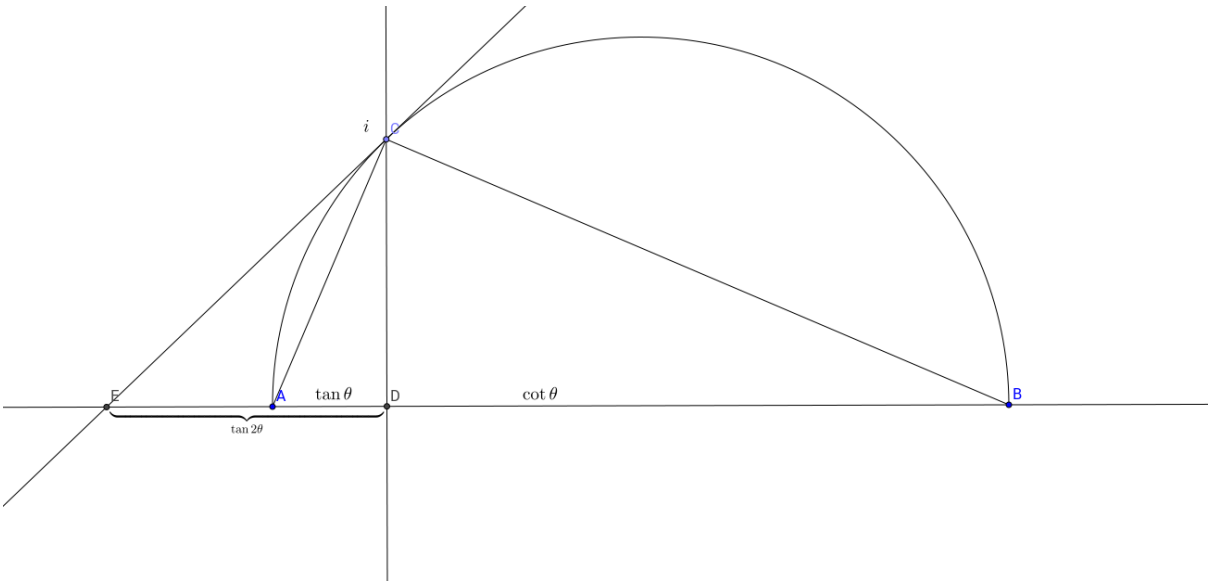
Hyperbolic Geometry Homework 2

Name: _____

1. (12 points) Verify that the following picture describes the effect of k_θ on the imaginary axis when $0 < \theta < \pi/2$. That is, verify that all the lengths and intersection points are correct. What is the slope of the tangent line? What is the angle formed between the imaginary axis and its image under k_θ?



2. (7 points) Compute the hyperbolic area of the triangle with vertices $-\sqrt{3}+i$, $\sqrt{3}+i$, and ∞.

3. (8 points) Give a continuous, surjective group homomorphism $f : \mathbf{R} \to \mathrm{SO}_2(\mathbf{R})$ with kernel $2\pi\mathbf{Z}$. Conclude that f induces an isomorphism of topological groups (in particular, a homeomorphism and a group isomorphism) between $\mathbf{R}/2\pi\mathbf{Z}$ and $\mathrm{SO}_2(\mathbf{R})$. Hint: If you do not use the angle addition formulas at any point, you're probably doing this exercise incorrectly.